**In complete sentences, using proper English and mathematical notation,** state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

SCORE: _____ / 5 PTS

IF $f$ IS CONTINUOUS ON $[a, b]$
AND $g(x) = \int_a^x f(t)\,dt$
THEN $g'(x) = f(x)$ ON $(a, b)$

IF $f$ IS CONTINUOUS ON $[a, b]$
AND $F'(x) = f(x)$ ON $[a, b]$
THEN $\int_a^b f(x)\,dx = F(b) - F(a)$

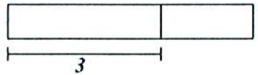
GRADED BY ME

IF $F'$ IS CONTINUOUS ON $[a, b]$.
THEN $\int_a^b F'(x)\,dx = F(b) - F(a)$

A rod is made of a mix of materials, so that its density is NOT constant along its length. **SCORE: _____ / 3 PTS**

Suppose the function $p(x)$ gives the linear density (in ounces per inch) of the rod $x$ inches from its left end.

For example, if $p(3) = 2$, that means that if the rod were made entirely of the material that is $3$ inches from the left end (the thin vertical sliver in the middle of the rod in the diagram), then each inch of that rod would weigh $2$ ounces.




What is the meaning of the equation $\int_1^3 p(x)\,dx = 8$ in this situation ?

GRADED BY ME

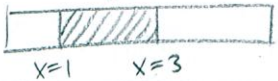
**NOTES:** **Your answer must use all three numbers from the equation, along with correct units.**
**Your answer should NOT use "x", "p(x)", "integral", "antiderivative", "rate of change" or "derivative".**

IF $w(x)$ = WEIGHT OF ROD FROM LEFT END TO X INCHES FROM LEFT END

$$w'(x) = p(x)$$

$$\text{SO } \int_1^3 p(x)\,dx = \int_1^3 w'(x)\,dx = w(3) - w(1) = 8\ \frac{\text{OUNCES}}{\cancel{\text{INCH}}} \times \cancel{\text{INCH}}$$

8 OUNCES = WEIGHT OF ROD FROM 1 INCH FROM LEFT END TO 3 INCHES FROM LEFT END



Evaluate the following integrals, or explain why they can't be evaluated.

ALL ITEMS ① POINT EXCEPT OTHERWISE MARKED

**SCORE: _____ / 11 PTS**

[a] $\displaystyle\int_{-3}^{3} \frac{7y}{\sqrt{1+y^4}}\,dy$ $= 0$ $\frac{1}{2}$

CONTINUOUS $\frac{1}{2}$

$$\frac{7(-y)}{\sqrt{1+(-y)^4}} = -\frac{7y}{\sqrt{1+y^4}}$$

ODD

$\frac{1}{2}$

[b] $\displaystyle\int_{\frac{3\pi}{4}}^{\pi} \frac{\sin t \cos t}{\sqrt{1-\sin^4 t}}\,dt$

$u = \sin^2 t$

$t = \pi \longrightarrow u = 0$

$t = \frac{3\pi}{4} \longrightarrow u = \frac{1}{2}$

$$\frac{du}{dt} = 2\sin t \cos t$$

$$\frac{1}{2}\,du = \sin t \cos t\,dt$$

$$\int_{\frac{1}{2}}^{0} \frac{1}{2}\frac{1}{\sqrt{1-u^2}}\,du$$

$$= \frac{1}{2}\sin^{-1} u \Big|_{\frac{1}{2}}^{0}$$

$$= \frac{1}{2}\left(0 - \frac{\pi}{6}\right)$$

$$= -\frac{\pi}{12}$$

[c] $\displaystyle\int_{-1}^{1} \frac{\sqrt[3]{1-\ln|x|}}{x}\,dx$

DISCONTINUOUS @ $x=0$

FTC DOES NOT APPLY

$\frac{1}{2}$

[d] $\displaystyle\int \frac{(2-3\sqrt{r})^2}{5r^2}\,dr$

$$= \int \frac{4-12r^{\frac{1}{2}}+9r}{5r^2}\,dr$$

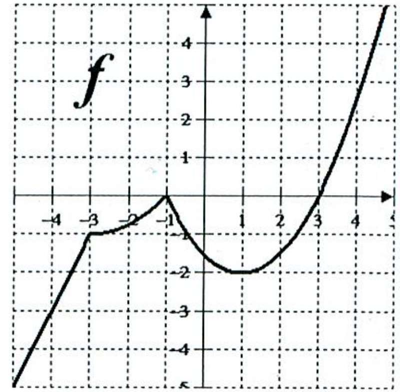
$$= \underbrace{\int \left(\frac{4}{5}r^{-2}-\frac{12}{5}r^{-\frac{3}{2}}+\frac{9}{5}r^{-1}\right)dr}$$

$$= -\frac{4}{5}r^{-1}-\frac{12}{5}(-2)r^{-\frac{1}{2}}+\frac{9}{5}\ln|r|+C$$

$$= \underbrace{-\frac{4}{5}r^{-1}+\frac{24}{5}r^{-\frac{1}{2}}+\frac{9}{5}\ln|r|}_{1\frac{1}{2}}+\underbrace{C}_{\frac{1}{2}}$$

$g'(x) = f(x)$

Let $g(x) = \int_{-3}^{x} f(t)\,dt$ , where $f$ is the function whose graph is shown on the right. **SCORE: _____ / 6 PTS**




[a] Write "I UNDERSTAND THAT THE GRAPH SHOWS $f$ , BUT THE QUESTIONS ASK ABOUT $g$ ".

[b] Find $g'(1)$ . **Explain your answer very briefly.**

$g'(1) = \underbrace{f(1) = -2}$ 1

[c] Find all critical numbers of $g$ . **Explain your answer very briefly.**

$g'(x) = \underbrace{f(x) = 0}_{1}$ @ $\underbrace{x = -1, 3}_{2}$ ← IF YOU GOT BOTH CORRECT ANSWERS, BUT ALSO ADDITIONAL INCORRECT ANSWERS, SUBTRACT 1 POINT PER INCORRECT ANSWER

[d] Find all intervals over which $g$ is concave down. **Explain your answer very briefly.**

$g'(x) = \underbrace{f(x) \text{ DECREASING}}_{1}$ ON $\underbrace{(-1, 1)}_{1}$

If $p(x) = \int_{3x}^{x^3} \sqrt{t^2 - 2}\, dt$, find $p'(2)$. **SCORE: _____ / 5 PTS**

$$p(x) = \int_{3x}^{2} \sqrt{t^2-2}\, dt + \int_{2}^{x^3} \sqrt{t^2-2}\, dt$$

$$= -\int_{2}^{3x} \sqrt{t^2-2}\, dt + \int_{2}^{x^3} \sqrt{t^2-2}\, dt$$

$$p'(x) = \frac{d}{d(3x)}\left(-\int_{2}^{3x} \sqrt{t^2-2}\, dt\right) \cdot \frac{d(3x)}{dx} + \frac{d}{d(x^3)} \int_{2}^{x^3} \sqrt{t^2-2}\, dt \cdot \frac{dx^3}{dx}$$

$$= \underbrace{-}_{1}\, \underbrace{\sqrt{(3x)^2-2}}_{1} \cdot \underbrace{3}_{1} + \underbrace{\sqrt{(x^3)^2-2}}_{\frac{1}{2}} \cdot \underbrace{3x^2}_{1}$$

$$p'(2) = -\sqrt{34} \cdot 3 + \sqrt{62} \cdot 12$$

$$= \underline{12\sqrt{62} - 3\sqrt{34}} \quad \frac{1}{2}$$